ASSIGNMENT I (1/2024)

040283211 ENGINEERING MATHEMATICS III (ENGLISH PROGRAM)

Instruction :

1. Assignment III has **eight questions.** Do below exercises individually. Write all questions and detailed solutions with your own handwriting.
2. Due date on August **2nd, 2024 before 23.59 pm.** Methods to submit based on agreement with lecturers of each section.

1). 1.1 Graph the cureve traced by the vector function $\vec{\mathbf{r}}(t)=\left\langle \sqrt{2}\sin t, \sqrt{2}\sin t, 2\cos t\right\rangle, 0\le t\le 2\pi$.

1.2 Find the vector function that describes the curve C of intersection between the surfaces $x^2+y^2-z^2=1,\ y=2x$ when $x=t$. Sketch the curve C.

1.3 Find $\vec{\mathbf{r}}'(t)$ and $\vec{\mathbf{r}}''(t)$ for the vector function $\vec{\mathbf{r}}(t)=t^2\,\hat{\mathbf{i}}+t^3\,\hat{\mathbf{j}}+\tan^{-1}t\,\hat{\mathbf{k}}$.

1.4 Evaluate the integral $\int\left(te^t\,\hat{\mathbf{i}}-e^{-2t}\,\hat{\mathbf{j}}+te^{t^2}\,\hat{\mathbf{k}}\right)dt$.

1.5 Find the length of the curve traced by $\vec{\mathbf{r}}(t)=e^t\cos(2t)\,\hat{\mathbf{i}}+e^t\sin(2t)\,\hat{\mathbf{j}}+e^t\,\hat{\mathbf{k}},\ 0\le t\le 3\pi$.

2). 2.1 Suppose that $\vec{\mathbf{r}}(t)=\left\langle t^2, t^3-2t, t^2-5t\right\rangle$ is the position vector of a moving particle. At what points does the particle pass through the $xy-$plane? What are its velocity and acceleration at these points?

2.2 The motion of a particle in space is described by $\vec{\mathbf{r}}(t)=a\cos t\,\hat{\mathbf{i}}+a\sin t\,\hat{\mathbf{j}}+bt\,\hat{\mathbf{k}},\ t\ge 0$, where a,b are nonzero constants.

(a) Compute $\|\vec{\mathbf{v}}(t)\|$.

(b) Compute $s=\int_0^t\|\vec{\mathbf{v}}(t)\|dt$ and verify that $\frac{ds}{dt}$ is the same as the result of part (a).

(c) Verify that $\frac{d^2s}{dt^2}\ne\|\vec{\mathbf{a}}(t)\|$.

3). 3.1 Find the unit tangent vector $\hat{\mathbf{T}}(t)$, unit normal vector $\hat{\mathbf{N}}(t)$, unit binormal vector $\hat{\mathbf{B}}(t)$, and curvature κ for a general circular helix described by $\vec{\mathbf{r}}(t)=a\cos t\,\hat{\mathbf{i}}+a\sin t\,\hat{\mathbf{j}}+bt\,\hat{\mathbf{k}}$, where a,b are nonzero constants.

3.2 Find the tangential and normal components of the acceleration at any t when the position vector is $\vec{\mathbf{r}}(t)=e^{-t}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$.

3.3 Find the curvature κ of an elliptical helix described by $\vec{\mathbf{r}}(t)=a\cos t\,\hat{\mathbf{i}}+b\sin t\,\hat{\mathbf{j}}+ct\,\hat{\mathbf{k}},$ where a,b,c are positive constants.

4). 4.1 Find the gradient of the function $F(x,y,z)=x^2z^2\sin(4y)$ at the point $(-2,\pi/3,1)$.

4.2 Find the directional derivative of the function $F(x,y,z)=x^2y^2(2z+1)^2$ at the point $(1,-1,1)$ in the direction $\langle 0,3,3\rangle$.

4.3 Find a vector that gives the direction in which the function $F(x,y,z)=\sqrt{xz}e^y$ decreases most rapidly at the point $(16,0,9)$. Find the minimum rate.

4.4 (a) If $f(x,y)=x^3-3x^2y^2+y^3$, find the directional derivative of f at a point (x,y) in the direction of $\hat{\mathbf{u}}=(1/\sqrt{10})(3\hat{\mathbf{i}}+\hat{\mathbf{j}})$.

(b) If $F(x,y)=D_{\hat{\mathbf{u}}}f(x,y)$ in part (a), find $D_{\hat{\mathbf{u}}}F(x,y)$.

4.5 The temperature T at a point (x,y,z) in space is inversely proportional to the square of the distance from (x,y,z) to the origin. It is known that $T(0,0,1)=500$. Find the rate of change of T at $(2,3,3)$ in the direction of $\langle 3,1,1\rangle$. In which direction from $(2,3,3)$ does the temperature T increase most rapidly? At $(2,3,3)$ what is the maximum rate of change of T?

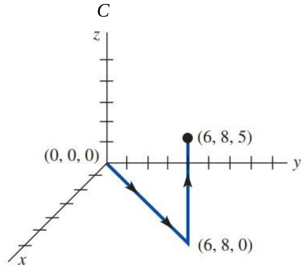
5). 5.1 Given $\vec{\mathbf{F}}(x,y,z)=xye^x\,\hat{\mathbf{i}}-x^3yze^z\,\hat{\mathbf{j}}+xy^2e^y\,\hat{\mathbf{k}}$. Find $\operatorname{curl}\vec{\mathbf{F}}$ and $\operatorname{div}\vec{\mathbf{F}}$.

5.2 Given that $\vec{\mathbf{F}}$ has continuity of all partial derivatives. Verify that $\operatorname{div}\left(\operatorname{curl}\vec{\mathbf{F}}\right)=0$.

5.3 Find $\operatorname{curl}\left(\operatorname{curl}\vec{\mathbf{F}}\right)$ for the vector field $\vec{\mathbf{F}}=xy\,\hat{\mathbf{i}}+4yz^2\,\hat{\mathbf{j}}+2xz\,\hat{\mathbf{k}}$.

6). 6.1 Let $G(x,y,z)=4xyz$ and the curve $C: x=\dfrac{t^3}{3}, y=t^2, z=2t, 0\le t\le 1$. Evaluate $\int_C G(x,y,z)\,ds$.

6.2 Evaluate $\int_C y\,dx+z\,dy+x\,dz$ where C is the curve from $(0,0,0)$ to $(6,8,5)$ as shown below.



6.3 Find the work done by the force $\vec{\mathbf{F}}(x,y,z)=yz\,\hat{\mathbf{i}}+xz\,\hat{\mathbf{j}}+xy\,\hat{\mathbf{k}}$ acting along the curve $\vec{\mathbf{r}}(t)=t^3\,\hat{\mathbf{i}}+t^2\,\hat{\mathbf{j}}+t\,\hat{\mathbf{k}}$ from $t=1$ to $t=3$.

7). 7.1 Show that $\displaystyle\int_{(1,1,\ln 3)}^{(2,2,\ln 3)} e^{2z}\,dx+3y^2\,dy+2xe^{2z}\,dz$ is independent of the path and evaluate the integral.

7.2 Given the vector function $\vec{\mathbf{F}}(x,y,z)=(2-e^z)\,\hat{\mathbf{i}}+(2y-1)\,\hat{\mathbf{j}}+(2-xe^z)\,\hat{\mathbf{k}}$ and the curve C is described by $\vec{\mathbf{r}}(t)=t\,\hat{\mathbf{i}}+t^2\,\hat{\mathbf{j}}+t^3\,\hat{\mathbf{k}}$ from $(-1,1,-1)$ to $(2,4,8)$. Evaluate $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$.

7.3 Given the force $\vec{\mathbf{F}}(x, y, z) = 8xy^3z\,\hat{\mathbf{i}} + 12x^2y^2z\,\hat{\mathbf{j}} + 4x^2y^3\,\hat{\mathbf{k}}$.

(a) Show that $\vec{\mathbf{F}}$ is conservative.

(b) Find the work done by $\vec{\mathbf{F}}$ acting along the helix $\vec{\mathbf{r}}(t) = 2\cos t\,\hat{\mathbf{i}} + 2\sin t\,\hat{\mathbf{j}} + t\,\hat{\mathbf{k}}$ from $(2,0,0)$ to $(1, \sqrt{3}, \pi/3)$.

8). 8.1 Use Green's theorem to evaluate the line integral $\oint_C xy^2\,dx + 3\cos y\,dy$, where C is the boundary of the region in the first quadrant determined by the graphs of $y = x^2$ and $y = x^3$.

8.2 Let R be the region bounded by a piecewise-smooth simple closed curve C.

Prove that $\frac{1}{2}\oint_C -y\,dx + x\,dy =$ area of R.

8.3 Use Green's theorem to find work done by the force $\vec{\mathbf{F}} = (x-y)\,\hat{\mathbf{i}} + (x+y)\,\hat{\mathbf{j}}$ around the closed curve as shown below.