ASSIGNMENT II (1/2024)

040283211 ENGINEERING MATHEMATICS III (ENGLISH PROGRAM)

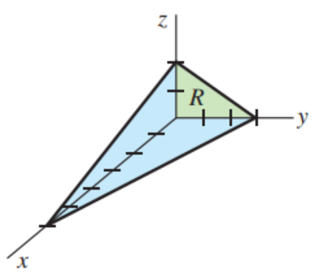
Instruction :

1. Assignment III has **five questions.** Do below exercises individually. Write all questions and detailed solutions with your own handwriting.
2. Due date on August **25th, 2024 before 23.59 pm.** Methods to submit based on agreement with lecturers of each section.

1). 1.1 Find the surface area of those portions of the sphere $x^2+y^2+z^2=2$ that are within the cone $z^2=x^2+y^2$.

1.2 Evaluate the surface integral $\iint_S G(x,y,z)\,dS$, where $G(x,y,z)=x+y+z$ and S is the surface of cone $z=\sqrt{x^2+y^2}$ between $z=1$ and $z=4$.

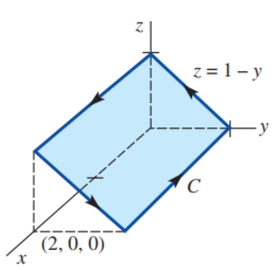
1.3 Evaluate $\iint_S (3z^2+4yz)\,dS$, where S is that portion of the plane $x+2y+3z=6$ in the first octant. Use the projection R of S onto the coordinate plane indicated in the figure below.



1.4 Let $\vec{\mathbf{F}}=e^y\,\hat{\mathbf{i}}+e^x\,\hat{\mathbf{j}}+18y\,\hat{\mathbf{k}}$ and the oriented upward surface S is that portion of the plane $x+y+z=6$ in the first octant. Find the flux of $\vec{\mathbf{F}}$ through the surface S.

2). 2.1 Assume that the surface S is oriented upward and that portions of the plane $2x+y+2z=6$ in the first octant. Verify Stokes' theorem for the vector field $\vec{\mathbf{F}}=z\,\hat{\mathbf{i}}+x\,\hat{\mathbf{j}}+y\,\hat{\mathbf{k}}$.

2.2 Let $\vec{\mathbf{F}}=z^2y\cos(xy)\,\hat{\mathbf{i}}+z^2x(1+\cos(xy))\,\hat{\mathbf{j}}+2z\sin(xy)\,\hat{\mathbf{k}}$ and C is the boundary of the plane $z=1-y$ shown in the figure below. Use Stokes' theorem to evaluate $\oint_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$.



2.3 Use Stokes' theorem to evaluate $\iint_S (\text{curl}\,\vec{\mathbf{F}})\cdot\hat{\mathbf{n}}\,dS$, where $\vec{\mathbf{F}} = y\,\hat{\mathbf{i}} + (y-x)\,\hat{\mathbf{j}} + z^2\,\hat{\mathbf{k}}$ and S is oriented upward and that portion of the sphere $x^2+y^2+(z-4)^2=25$ for $z \geq 0$.

3). 3.1 Use the divergence theorem to find the outward flux $\oiint_S (\vec{\mathbf{F}}\cdot\hat{\mathbf{n}})\,dS$, where $\vec{\mathbf{F}} = y^3\,\hat{\mathbf{i}} + x^3\,\hat{\mathbf{j}} + z^3\,\hat{\mathbf{k}}$ and S is the surface of the region bounded within by $z=\sqrt{4-x^2-y^2}$, $x^2+y^2=3$ and $z=0$.

3.2 The electric field at a point $P(x,y,z)$ due to a point charge q located at the origin is given by the inverse square field $\vec{\mathbf{E}} = q\dfrac{\vec{\mathbf{r}}}{\|\vec{\mathbf{r}}\|^3}$, where $\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$. Suppose S is a closed surface, S_a is a sphere $x^2+y^2+z^2=a^2$ lying completely within S, and D is the region bounded between S and S_a as shown in the figure below. Show that the outward flux of $\vec{\mathbf{E}}$ for the region D is zero.

3.3 Assume that the surface S forms the boundary of a closed and bounded region D.
If $\vec{\mathbf{F}} = P\,\hat{\mathbf{i}} + Q\,\hat{\mathbf{j}} + R\,\hat{\mathbf{k}}$ and $P, Q,$ and R have continuous second partial derivatives, show that $\oiint_S (\text{curl}\,\vec{\mathbf{F}}\cdot\hat{\mathbf{n}})\,dS = 0$.

4). 4.1 Let A be a ninzero 4×6 matrix.

(a) What is the maximum rank that A can have?

(b) If $\text{rank}(\mathrm{A}|\vec{\mathbf{B}}) = 2$, then for what value(s) of $\text{rank}(\mathrm{A})$ is the system $\mathrm{A}\vec{\mathbf{X}} = \vec{\mathbf{B}},\ \vec{\mathbf{B}} \neq \vec{\mathbf{0}},$ inconsistent? Consistent?

(c) If $\text{rank}(\mathrm{A}) = 3$, then how many parameters does the solution of the system $\mathrm{A}\vec{\mathbf{X}} = \vec{\mathbf{0}}$ have?

4.2 If the augmented matrix of a system of linear equations is

$$[\mathrm{A}|\vec{\mathbf{B}}] = \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & c^2-9 & c+3 \end{array}\right].$$

Find for what value(s) of c the system has

(a) a unique solution.

(b) infinitely many solutions.

(c) no slution.

4.3 Let $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -2 & 3 & a \\ 3 & -1 & b & 2 \end{array}\right]$ be the augmented matrix for a system of linear equations. Find for what values of a and b the system has

(a) a unique solution.

(b) infinitely many solutions.

(c) no slution.

4.4 Use elementary row operations to determine the solution of the following linear system.

$$\begin{aligned} &2x - y + 3z + 4w = 9 \\ &x \qquad\ - 2z + 7w = 11 \\ &3x - 3y + z + 5w = 8 \\ &2x + y + 4z + 4w = 10 \end{aligned}$$

5). Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the eigenvalues and corresponding eigenvectors of A.
