# ASSIGNMENT I

## 040283101 MATHEMATICS I (English Programme)

**Instruction:**

1. Assignment I has nine questions. Do below exercise individually.
2. Allow writing by pen or pencil on paper, iPad or Tablet.
3. Due date on Sunday 25 August 2024 before 23.59 pm.

1. Find the given limits. **(Do not use L'Hopital's Rule)**

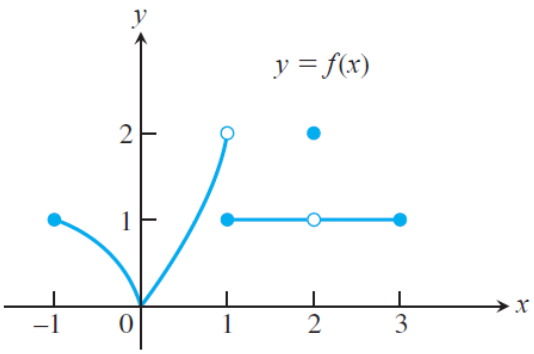
1.1) $\lim\limits_{x\to 1} \dfrac{xe^x - e^x}{x^2 - 1}$

1.2) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\sin 4x}$

1.3) $\lim\limits_{x\to \infty} \left(\sqrt{x^2+5x} - x\right)$

1.4) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{9x^2+6}}{5x-1}$

2. Consider the given graph of function $f$.




2.1) Find each limit, or explain why it does not exist.

a) $\lim\limits_{x\to 0} f(x)$

b) $\lim\limits_{x\to 1} f(x)$

c) $\lim\limits_{x\to 2} f(x)$

2.2) State, with reasons, all values of $x$ at which $f$ is discontinuous.

2.3) Use the Squeeze theorem to evaluate $\lim\limits_{x\to 2} f(x)$ where $2x-1 \le f(x) \le x^2 - 2x + 3$.

Hint: Suppose $f, g$ and $h$ are functions for which $g(x) \le f(x) \le h(x)$ for all $x$.

If $\lim\limits_{x\to a} g(x) = L$ and $\lim\limits_{x\to a} h(x) = L$ then $\lim\limits_{x\to a} f(x) = L$.

3. 3.1) Find a number $k$ so that

$$f(x)=\begin{cases} cx^2+2x, & x<2 \\ x^3-cx, & x\geq 2 \end{cases}$$

is continuous at $x=2$.

3.2) Find the slope of the tangent line to the graph of $f(x)=x^2-3x$ at $x=3$ by using the **definition of derivative**.

4. 4.1) Find the derivative of the given function.

a) $y=x^3+2x-\dfrac{1}{x}$

b) $y=(7x-1)^3$

c) $y=\dfrac{5^x}{1+x}$

d) $y=e^{\cosh(3x)}$

e) $y=x^2\tan(3x)$

4.2) Let $y=\ln(x^2)$. Find the following derivatives:

a) $\dfrac{dy}{dx}$ b) $\dfrac{d^2y}{dx^2}$

c) $\dfrac{d^3y}{dx^3}$ d) $\dfrac{d^4y}{dx^4}$

e) $\dfrac{d^{100}y}{dx^{100}}$

5. 5.1) Suppose $y=5u^2+1,\ u=\sqrt{4t+1}$ and $t=x^2+1$. Find $\dfrac{dy}{dx}$ at $x=1$.

5.2) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ where $y=\dfrac{(2x-1)^{\sqrt{x}}}{(x+1)^2(5x+4)}$.

6. Given the following equation

$$3x^2+2xy^2-e^x+y=0.$$

6.1) Find the point $P(x,y)$ at which the graph intercepts y-axis.

6.2) Find the slope of the tangent line of the graph at the point $P$.

6.3) Find the equation of tangent line of the graph at the point $P$.

7. The curve is given by $x = e^{2t}, \quad y = t^3 - 2t$.

7.1) Find $\frac{dy}{dx}$.

7.2) Find $\frac{d^2y}{dx^2}$.

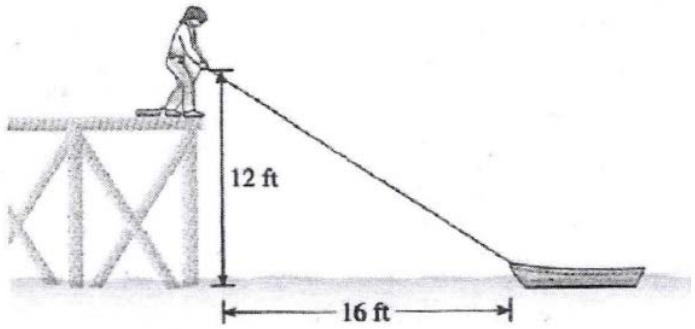
7.3) Find an equation of the tangent line and equation of normal line at the point corresponding to $t = 0$.

8. 8.1) Let $s(t) = t^3 - 4t$ be a position function of a particle that moves on a horizontal line when $t \geq 0$.

a) What is the acceleration of the particle when $v(t) = 2$?

b) What is the position of the particle when $a(t) = 18$?

8.2) A person is standing at the end of a pier 12 ft above the water level and is pulling in a rope attached to a rowboat at the water line at the rate of 6 feet/minute. How fast is the boat moving when it is 16 ft from the pier?




9. Use **L'Hopital's rule** to find the limit, or state that it does not exist.

9.1) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\ln(\sin x)}{(2x - \pi)^2}$

9.2) $\lim\limits_{x \to 0^+} \left(\frac{1}{x}\right)^x$